office hour week 12 (a)

First Name: _____

Last Name: _____

Student Number: _____

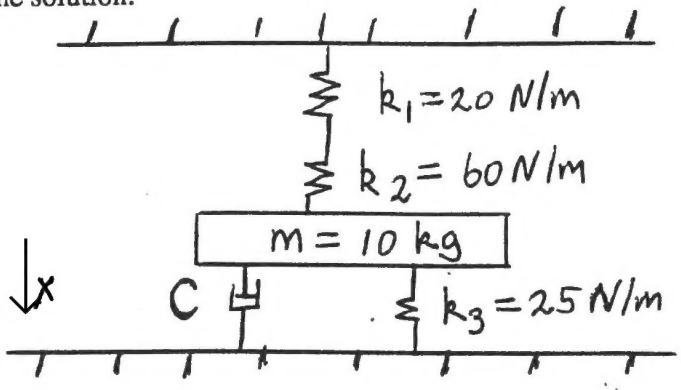
MIE 200F – Test No. 9 – November 30, 1998
Test Duration – 20 minutes

(a) What is the natural frequency ω_n of the system if the damper C = 0?

(b) What damper value C is required to reduce the vibration frequency by about 2%?

The equation: $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$ has the solution:

$x(t) = C_1 \sin(\omega_d t + \phi) \exp(-\zeta\omega_n t)$,
where C_1 and ϕ are unknown constants,
and the damped frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$



$K_{total} = k_3 + \frac{k_1 k_2}{k_1 + k_2}$

$= 25 + \frac{(20)(60)}{20+60}$

$= 25 + 15$

$= 40$ N/m

(a) $F = -40x = m\ddot{x}$

$10\ddot{x} + 40x = 0$

$\ddot{x} + \omega_n^2 x = 0 \longleftrightarrow \ddot{x} + 4x = 0$

$\omega_n^2 = 4 \Rightarrow$ **$\omega_n = 2\ s^{-1}$**

(b) $\frac{\omega_d}{\omega_n} = \sqrt{1-\zeta^2} = 0.98 \Rightarrow \zeta = 0.199$

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0 \Leftrightarrow \ddot{x} + \frac{c}{m}\dot{x} + 4x = 0$

$\Rightarrow 2\zeta\omega_n = c/m$

$(2)(0.199)(2) = c/10$

$c = 7.96\ \frac{N}{m/s}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

First Name: _____

Student Number: _____

office hour week 12 (b)

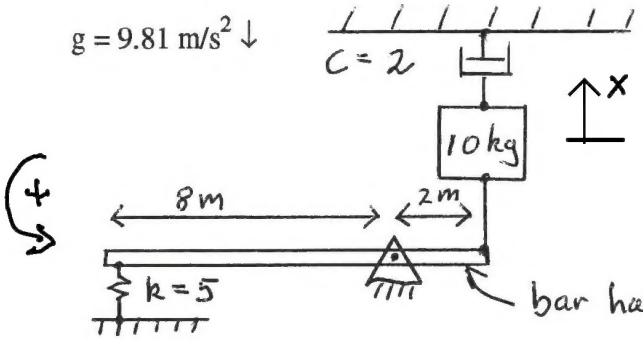
MIE 200F - Quiz number 8a - November 28/01
quiz duration = 25 minutes

A block is supported by the system of dampers and springs as shown. Assume that the mass can move only in the vertical direction, with no rotation. Values of all dampers and springs are given in SI units.

(a) At what frequency, expressed in Hertz, will this system vibrate?

(b) How long will it take for the oscillations of this system to damp down to only 1% of their initial magnitude?

$g = 9.81\ m/s^2 \downarrow$



The equation $\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$, has the solution

$x(t) = C_1 \cos(\omega_d t)\exp(-\zeta\omega_n t) + C_2 \sin(\omega_d t)\exp(-\zeta\omega_n t)$

$= C_3 \sin(\omega_d t + \varphi)\exp(-\zeta\omega_n t)$

where $\omega_d = \omega_n (1-\zeta^2)^{1/2}$

(a) $\Sigma F = -2\dot{x} - (5)\left(\frac{8}{2}\right)^2 x = m\ddot{x}$

$10\ddot{x} + 2\dot{x} + 80x = 0$

$\ddot{x} + 0.2\dot{x} + 8x = 0 \iff \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$

$\Rightarrow \omega_n = \sqrt{8}$; $2\zeta\omega_n = 0.2 \Rightarrow \zeta = 0.0354$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.83\ s^{-1}$

$f = \omega_d / 2\pi = 0.45\ Hz$

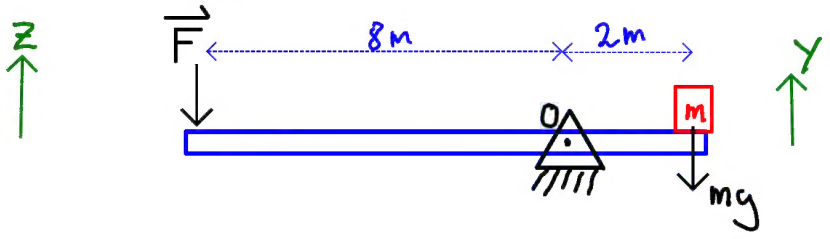
(b) $\exp(-\zeta\omega_n t) = 0.01$

$\exp(-[.035]\sqrt{8}\,t) = 0.01$

take logs of both sides:

$0.035\sqrt{8}\,t = 4.605$

$t = 46.5$ seconds



$\sum M_O = 0$

$\vec{F} = \frac{2}{8} m\vec{g}$

$\Delta z = -\frac{8}{2}\Delta y$

office hour week 12(c)

Dynamics Final exam 2002

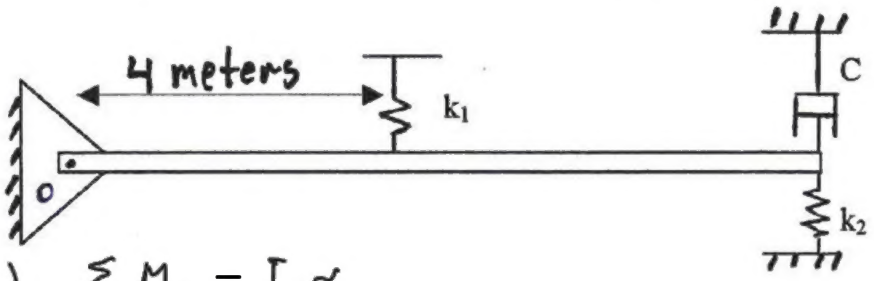
3. A long thin beam of mass 12 kg and length 10 meters is pinned at one end to a wall as shown. The spring k_1 has a stiffness of 60 N/m, and spring k_2 has a stiffness of 40 N/m. The beam is in equilibrium when it is in a horizontal orientation as show.

   (a) Write the second order differential equation for this system in terms of C, k_1, and k_2, to describe small angular oscillations about the equilibrium point, .

   (b) Find the natural frequency of vibration ω_n of the beam, if the damper C is set to zero. Assume that the amplitude of vibration is very small.

   (c) When it vibrates with the damper C set to zero, the maximum angular deflection of the beam from its equilibrium position is 0.015 radians. What is the maximum angular acceleration α experienced by the beam under these conditions?

   (d) A damper C is now added at the position shown to reduce the vibration amplitude by 50% in 6 seconds. What is the damping ratio? What is the value of C?



+↻

$g = 9.81\ m/s^2$ ↓

(a) $\sum M_o = I_o \alpha$

$$-(4\theta)(k_1)(4) - (10\theta)(k_2)(10) - (10\dot{\theta})(C)(10) = \left(\tfrac{1}{12}mL^2 + m\left(\tfrac{L}{2}\right)^2\right)\ddot{\theta}$$

$$\rightarrow \boxed{\tfrac{1}{3}mL^2\ddot{\theta} + 100C\dot{\theta} + (16k_1 + 100k_2)\theta = 0}$$

(b) $400\ddot{\theta} + 100C\dot{\theta} + 4960\theta = 0$ (C → 0)

$$\rightarrow \ddot{\theta} + 12.4\theta = 0 \rightarrow \boxed{\omega_n = \sqrt{12.4} = 3.5\ s^{-1}}$$

(c) $\theta = A\sin(\omega_n t + \phi)$ (A → 0.015 meters) $\rightarrow \dot{\theta} = (\omega_n)(0.015)(\cos(\omega_n t + \phi))$

$$\rightarrow \ddot{\theta} = -(\omega_n)^2(0.015)(\sin(\omega_n t + \phi))$$

$$\rightarrow \boxed{\ddot{\theta}_{max} = (\omega_n)^2(0.015) = (12.4)(0.015) = 0.186\ \text{radians}}$$

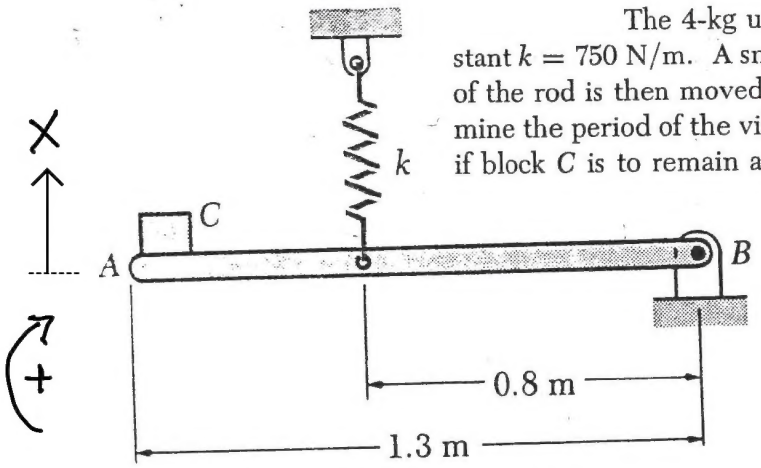
(d) $\ln\left\{\exp(-\zeta\omega_n t)\Big|_{t=6} = 0.5\right\}$

$$-(\zeta)(3.5)(6) = -0.693$$

$$\boxed{\zeta = 0.033}$$

$\ddot{\theta} + 0.4C\dot{\theta} + 12.4\theta = 0 \leftrightarrow \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0 \rightarrow C$

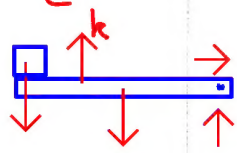
Some Pre-exam Thoughts...



The 4-kg uniform rod AB is attached as shown to a spring of constant $k = 750$ N/m. A small 0.5-kg block C is placed on the rod at A. (*a*) If end A of the rod is then moved ~~down~~ up through a small distance δ_0 and released, determine the period of the vibration. (*b*) Determine the largest allowable value of δ_0 if block C is to remain at all times in contact with the rod.

Two-dimensional mass is rotating about fixed point
$\Rightarrow$ use $\Sigma M_0 = I_0 \alpha$, where fixed point "O" is point B.

$$I_0 = (I_0)_{AB} + (I_0)_C = \underbrace{\left(\tfrac{1}{12}\right)(4\text{kg})(1.3\text{ m})^2 + (4\text{kg})(.65\text{m})^2}_{AB} + \underbrace{(.5)(1.3\text{m})^2}_{C}$$
$$= 3.1 \text{ kg m}^2$$



+ Displace a small angle θ from equilibrium.
$\Rightarrow$ Only moment that significantly changes is from Spring.
Spring is shortened by a distance $(0.8)\theta$
$\Rightarrow$ Force in spring decreases by $0.8\theta k$
i.e. net change is a downward increase of $0.8\theta k$
$\Rightarrow$ This leads to a negative moment of $(0.8)(0.8\theta k)$
$\Rightarrow$ Total moment $= -0.8^2\theta k = I_0\ddot{\theta}$

$\Rightarrow 3.1\ddot{\theta} + (480)\theta = 0 \Rightarrow \ddot{\theta} + 155\theta = 0 \leftrightarrow \ddot{\theta} + \omega_n^2\theta = 0$

$\omega_n^2 = 155$, $\omega_n = 12.45$

$$\Rightarrow \boxed{\theta = c_1 \sin(12.45t) + c_2 \cos(12.45t)}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.505 \text{ s}$$

but movement of block $C = 1.3\,\theta$

$\Rightarrow x = 1.3c_1 \sin(12.45t) + 1.3c_2 \cos(12.45t)$

initial conditions: $x(t=0) = \delta_0$, $\dot{x}(t=0) = 0$

$\Rightarrow c_1 = 0$, $c_2 = \delta_0/1.3$

$\Rightarrow x = \delta_0 \cos(12.45t)$, $\dot{x} = -12.45\delta_0 \sin(12.45t)$,
$\ddot{x} = -12.45^2 \delta_0 \cos(12.45t)$

set $(12.45^2)(\delta_0) = 9.81 \Rightarrow$ $\boxed{\delta_0 = 0.063 \text{ m}}$